Ordered Patch Theory

Appendix T-5: Constants Recovery — Structural Bounds from R(D) Optimisation

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Original Task T-5: Constants Recovery Problem: Standard physics treats dimensionless constants as brute facts. Under OPT, these constants should emerge as optimal solutions to the rate-distortion optimization problem at the observer boundary. **Deliverable:** Constraints or bounds on dimensionless constants from $C_{\max}$ limits.

Closure status: T-5a PARTIALLY RESOLVED; T-5b PARTIALLY RESOLVED (Heuristic limitations). This appendix assesses the formal constraint derivations required by OPT. Four distinct elements are mapped. T-5a.1: Using standard physics constants as inputs, the Stability Filter structurally aligns the codec's length scale to roughly the Planck length ($l_{\text{codec}} \approx 1.67 l_P$) if assuming a binary alphabet ($q = 2$). T-5a.2: an upper bound on Λ from the de Sitter temperature. T-5b.1: a heuristic ansatz mapping a lower bound on α to the cognitive quantum h^*. T-5b.2: an upper bound on G from cognitive timescale stability. The honest limitation: OPT's constraints are necessary boundary heuristic checks — they rule out vast regions of parameter space but do not precisely derive scalar values from first principles.

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§1. Inputs from T-1 through T-4

T-5 is the convergence point of the four preceding appendices. The following results are available as starting conditions.

Source	Result used in T-5	Value
T-1 (R(D))	Cognitive quantum $h^* = C_{\max} \cdot \Delta t$	0.5–0.8 bits/moment
T-1	Rate-distortion lower bound: $R_{T,h}(D) \geq E_{T,h}(\nu) - D$	T-1 §2.3

Source	Result used in T-5	Value
T-2 (Entropic gravity)	$G_{\text{OPT}} = c^2_{\text{codec}}/\log_2 q$	Identified with G conditionally via structural limits
T-2	$c_{\text{codec}} = c$, $\hbar_c = \hbar$	Standard values
T-3 (MERA/RT)	$S_{\text{render}} \leq \lvert\partial A\rvert \log q$ (area law)	$\log q$ bits per Planck area
T-4 (MDL)	$K(\text{IC} \mid \text{SP}) \approx 300$ bits; $K_0 \approx 36$ bits	Order-of-magnitude
Preprint §3.9	Fano bound on substrate identification	$P(\text{error}) \geq 1 - (T \cdot C_{\max} + 1)/\log N$

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§2. The Planck Scale Order-of-Magnitude Alignment — Theorem T-5a.1

Combining T-2's gravitational parameter requirements with T-3's structural area-laws yields an order-of-magnitude structural mapping bridging standard SI scales with natural codec variables.

2.1 Setup: Entropic Consistency Requirements

From T-2 §4.5, resolving conditional metric equivalence explicitly defers to resolving a formal dimensional bits-to-mass mapping parameter α. Factoring explicitly dimensionally tracking limits structurally frames:

$$G_{\text{OPT}} = \frac{c^2_{\text{codec}}}{\log_2 q} \tag{T-2}$$

Substituting $G_{\text{OPT}} = G$ and $c_{\text{codec}} = c$ into the definition of the Planck length $l_P^2 = G\hbar/c^3$ gives $l_P^2 = l^2_{\text{codec}}/\log_2 q$, hence $l^2_{\text{codec}} \propto l_P^2$.

From T-3, the absolute coding capacity of a boundary screen of area A is:

$$N_{\text{OPT}} = \frac{A}{l^2_{\text{codec}}} \cdot \log_2 q \tag{T-3}$$

The Bekenstein-Hawking entropy calculation derives dynamically in natural units that physical event horizons map at $A/(4l_P^2)$ nats. Converted directly to bits via $\ln 2$:

$$N_{\text{BH}} = \frac{A}{4l_P^2 \cdot \ln 2} \quad \text{bits}$$

2.2 Deriving the Scale Offset

We face two formal structural matching requirements mapping geometric equivalents mutually.

Condition A (Gravitational Mapping): Setting $G_{\text{OPT}} = G$ gives $l^2_{\text{codec}}/\log_2 q \equiv l^2_P$. For a minimal binary alphabet ($q = 2$, $\log_2 q = 1$), this yields:

$$l_{\text{codec}} = l_P$$

Condition B (Entropy Mapping): Setting $N_{\text{OPT}} = N_{\text{BH}}$ gives:

$$\frac{A}{l^2_{\text{codec}}} \cdot 1 = \frac{A}{4l^2_P \ln 2} \implies l_{\text{codec}} = 2\sqrt{\ln 2} \cdot l_P \approx 1.665\, l_P$$

2.3 Theorem T-5a.1 — Order-of-Magnitude Alignment

Theorem T-5a.1 (Planck Scale Consistency Check). The two matching conditions — gravitational (Condition A) and entropic (Condition B) — are mutually consistent only if $q = 4\ln 2 \approx 2.77$. For the conventional binary alphabet $q = 2$, they yield $l_{\text{codec}} = l_P$ and $l_{\text{codec}} \approx 1.67\, l_P$ respectively — differing by the factor $2\sqrt{\ln 2}$. Both values lie within a single order of magnitude of l_P, confirming structural alignment at the order-of-magnitude level.

Remark on the scale offset. The factor $2\sqrt{\ln 2}$ arises from the unit mismatch between OPT's binary convention and the Bekenstein-Hawking formula's natural convention. This is an internal consistency gap, not a rounding error; it is resolved when q is treated as a free parameter rather than fixed at 2. ■

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§3. Cosmological Constant Bound — Theorem T-5a.2

The Stability Filter requires the rendered spacetime to support a coherent observer. A de Sitter space with cosmological constant Λ generates a Gibbons-Hawking temperature T_{dS} that constitutes irreducible thermal noise in the codec's environment. If T_{dS} exceeds the energy scale of cognitive coherence, the Filter cannot maintain a stable patch.

3.1 Derivation

The de Sitter horizon temperature (Gibbons-Hawking 1977) is:

$$T_{\text{dS}} = \frac{\hbar c\sqrt{\Lambda/3}}{2\pi k_B}$$

The minimum energy of a cognitive update is set by Landauer's principle (preprint Eq. 10): each bit erasure at the codec costs at least $k_B T \ln 2$. The cognitive coherence energy per update is $\hbar \cdot C_{\max}$. The Stability Filter requires:

$$k_B T_{\mathrm{dS}} < \hbar C_{\max}$$

Substituting and solving for Λ:

$$\frac{\hbar c\sqrt{\Lambda/3}}{2\pi} < \hbar C_{\max} \implies \sqrt{\Lambda/3} < \frac{2\pi C_{\max}}{c}$$

Theorem T-5a.2 (Cosmological Constant Upper Bound). For the Stability Filter to maintain a coherent cognitive patch against de Sitter vacuum fluctuations:

$$\boxed{\Lambda \leq \frac{12\pi^2 C_{\max}^2}{c^2}}$$

For numerical evaluation, $C_{\max}$ should be expressed in nats/s when the formula is applied alongside $\hbar$ in SI units.

Numerically using standard proxy values: fixing $C_{\max} \approx 10$ bits/s ≈ 6.93 nats/s generates a conservative functional upper limit constraint of $\Lambda \leq 6.3 \times 10^{-15}$ m^{-2}. The observed value $\Lambda_{\mathrm{obs}} \approx 1.09 \times 10^{-52}$ m^{-2} satisfies this bound smoothly by approximately 37 full orders of magnitude. ■

Remark. OPT's Λ bound is weaker than standard anthropic bounds (structure formation requires $\Lambda \lesssim 10^{-121}$ in Planck units). The OPT bound is a necessary condition on the *cognitive* stability of the observer, not on cosmological structure formation. The margin of 37 orders between the bound and the observed value reflects the extraordinary smallness of Λ — consistent with OPT's prediction (preprint §8) that the de Sitter geometry is the Stability Filter's preferred ground state for branch separation.

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§4. Fine-Structure Constant Lower Bound — Theorem T-5b.1

This is T-5's most novel result: a lower bound on α derived entirely from OPT's internal parameters — specifically from the cognitive quantum $h^* = C_{\max} \cdot \Delta t$ established in T-1 and the biological temperature scale T_{bio}.

4.1 The Codec Discriminability Ansatz Condition

The observer's codec must dynamically isolate atomic binding levels as distinct resolvable states — otherwise complex structural chemistry vanishes from the codec's descriptive capability limit.

We postulate a structural codec discriminator ansatz requiring that binding energies exceed thermal fluctuations by a divergence factor $f(h^*)$ that scales inversely with available bandwidth:

$$E_{\text{binding}}(\alpha, n) \geq k_B T_{\text{bio}} \cdot f(h^*)$$

To practically bound the constraints, we must select an illustrative heuristic form for $f(h^*)$. A natural candidate reflecting the exponential difficulty of resolving discrete quantum states under extreme codec bandwidth limitation is $f(h^*) = 2^{1/h^*}$. This specific ansatz explicitly diverges as $h^* \to 0$ (forcing chemical contrast requirements to infinity for a zero-bandwidth observer).

Note: The resulting numerical lower bound on α is highly sensitive to this chosen contrast function form $f(h^*)$. We use $2^{1/h^*}$ to demonstrate the existence of the bound, while acknowledging formal derivation of the true $f(h^*)$ from Shannon capacity limits is deferred.

For our illustrative heuristic $2^{1/h^*}$, assuming $h^* = 0.5$ bits: $2^{1/h^*} = 4.0$. For $h^* = 0.8$ bits: ≈ 2.38.

The relevant binding energy for chemical complexity occurs at the first bonding orbital ($n = 2$):

$$E_{\text{binding}}(\alpha, n = 2) = \frac{\alpha^2 m_e c^2}{8}$$

Substituting into the discriminability ansatz condition yields:

$$\frac{\alpha^2 m_e c^2}{8} \geq k_B T_{\text{bio}} \cdot 2^{1/h^*}$$

4.2 Theorem T-5b.1

Theorem T-5b.1 (Fine-Structure Constant Heuristic Ansatz Lower Bound). Applying the specific exponential heuristic discriminator ansatz $f(h^*) = 2^{1/h^*}$, for the Stability Filter to physically secure a chemically complex stream, empirical parameters map the constraint securely:

$$\boxed{\alpha \geq \alpha_{\min}(f) \approx \sqrt{\frac{8\, k_B T_{\text{bio}} \cdot 2^{1/h^*}}{m_e c^2}}}$$

Numerically ($T_{\text{bio}} = 310$ K, $h^* = 0.5$ bits, $m_e c^2 = 511$ keV):

$$\alpha_{\min} = \sqrt{\frac{8 \times (1.381 \times 10^{-23}) \times 310 \times 4.0}{(9.109 \times 10^{-31}) \times (2.998 \times 10^{8})^2}} \approx 1.29 \times 10^{-3}$$

The observed $\alpha_{\text{obs}} = 1/137.036 \approx 7.30 \times 10^{-3}$ satisfies $\alpha_{\text{obs}}/\alpha_{\min} \approx 5.6$ — safely above the bound, with a margin of factor ~5.6. For $h^* = 0.8$ bits: $\alpha_{\min} \approx 9.97 \times 10^{-4}$, giving a margin of factor ~7.3. ■

4.3 Physical Interpretation

The bound $\alpha_{\min} \approx \sqrt{8k_B T_{\text{bio}} \cdot 2^{1/h^*}/(m_e c^2)}$ reveals a structural relationship: the electromagnetic coupling constant is bounded from below by a combination of the *cognitive bandwidth* (via h^*), the *thermal environment* (via T_{bio}), and the *electron rest mass* (via $m_e c^2$). Standard anthropic arguments bound α from below via the requirement that atoms exist, but do not connect this to $C_{\max}$. OPT does.

The bound also shows why $C_{\max}$ must satisfy a joint constraint with α: if $C_{\max}$ were reduced by a factor of 10 ($h^* = 0.05$ bits), then $2^{1/h^*} = 2^{20} \approx 10^6$, and $\alpha_{\min} \approx 0.3$, far exceeding the actual α. A universe with our α and a dramatically lower $C_{\max}$ would fail the Stability Filter — chemistry would be unresolvable in the available cognitive bandwidth.

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§5. Gravitational Stability Constraint — Theorem T-5b.2

The standard Newtonian gravitational free-fall collapse timescale of a structure of mass M and radius R is $t_{\text{collapse}} = \sqrt{R^3/(GM)}$. For the codec to maintain a coherent narrative of its own physical substrate, this bounding limit timescale must exceed the cognitive update interval Δt.

(Note: The free-fall timescale is a strictly conservative geometric proxy bounding structural stability. The true condition securely depends upon electromagnetic vs gravitational structural force limits formally yielding tighter bounds natively.)

Theorem T-5b.2 (Gravitational Stability Bound). The Stability Filter requires that the observer's physical substrate not gravitationally collapse on the cognitive timescale. For a substrate of mass M_{obs} and radius R_{obs}:

$$\boxed{G < \frac{R_{\text{obs}}^3}{M_{\text{obs}}\,\Delta t^2}}$$

For a human brain ($R_{\text{obs}} = 0.07$ m, $M_{\text{obs}} = 1.4$ kg, $\Delta t = 0.05$ s):

$$G < \frac{(0.07)^3}{1.4 \times (0.05)^2} = 9.8 \times 10^{-2}\ \text{m}^3\text{kg}^{-1}\text{s}^{-2}$$

The observed $G = 6.67 \times 10^{-11}$ satisfies this by 10 orders of magnitude. ■

The complementary bound, from T-2 §7.1: the observer's Schwarzschild radius must be enormously smaller than the observer's physical radius (the codec must not be inside its own event horizon):

$$r_S(M_{\text{obs}}) = \frac{GM_{\text{obs}}}{c^2} \approx 1.04 \times 10^{-27}\ \text{m} \ll R_{\text{obs}} \approx 0.07\ \text{m} \quad [\text{by 25 orders}]$$

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§6. The Complete Constraint Picture

Constant	OPT constraint	OPT expected scalar	Observed	Margin	Source
q (alphabet)	Assume min binary $q = 2$	$q = 2$	N/A	Input assumed	T-5a.1
l_{codec}	Structural mapping	$\approx 2.7 \times 10^{-35}$ m	$l_P \approx 1.6 \times 10^{-35}$ m	$\approx 1.67 \times l_P$	T-5a.1
$c, \hbar, G$	Empirical inputs required	Standard values	CODATA values	N/A	T-5a
Λ	Upper bound limit	$\leq 6.3 \times 10^{-15}$ m^{-2}	1.09×10^{-52} m^{-2}	$10^{37}\times$ below	T-5a.2
α	Heuristic lower bound	$\geq 1.29 \times 10^{-3}$	7.30×10^{-3}	5.6× above	T-5b.1
G	Upper bound limit	$< 9.80 \times 10^{-2}$ m^{3}kg^{-1}s^{-2}	6.67×10^{-11}	$10^{9.2}\times$ below	T-5b.2
α_G/α	$\alpha_G \ll \alpha$ (hierarchy)	$\alpha_G/\alpha \leq 1$	4.2×10^{-43}	Hierarchy confirmed	T-5b.2

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§7. The Joint $C_{\max}$–α Constraint Surface

Theorem T-5b.1 reveals a joint constraint between α and $C_{\max}$ that goes beyond individual bounds. Rearranging the lower bound:

$$\frac{\alpha^2 m_e c^2}{8 k_B T_{\text{bio}}} \geq 2^{1/h^*} = 2^{1/(C_{\max}\Delta t)}$$

Taking logarithms of both sides and solving for $C_{\max}$:

$$C_{\max} \geq \frac{1}{\Delta t \cdot \log_2\left(\dfrac{\alpha^2 m_e c^2}{8 k_B T_{\text{bio}}}\right)}$$

This is a **joint constraint surface** in the $(\alpha, C_{\max})$ plane — a hyperbola. For any given α, it provides a lower bound on $C_{\max}$ (the observer must have sufficient cognitive bandwidth to resolve chemical discriminability); equivalently, for any given $C_{\max}$, it provides a lower bound on α.

Verifying our universe at ($\alpha = 1/137, C_{\max} = 10$ bits/s):

$$C_{\max}^{\min}(\alpha = 1/137) = \frac{1}{0.05 \cdot \log_2\left(\frac{(7.3\times10^{-3})^2\times511\text{ keV}}{8\times26\text{ meV}}\right)} \approx \frac{1}{0.05 \times 10.0} = 2.0\text{ bits/s}$$

The observed $C_{\max} \approx 10$ bits/s places us comfortably *above* the minimum threshold (the bound at the discriminability threshold would be 2 bits/s; we operate well above it). The allowed region satisfies both:

- $\alpha \geq \alpha_{\min}(C_{\max})$: chemistry is resolvable in the cognitive bandwidth
- $C_{\max} \geq C_{\max}^{\min}(\alpha)$: cognitive bandwidth is sufficient to resolve chemical discriminability at the given α

Note: a *separate* selection-pressure argument suggests that extremely high $C_{\max}$ would trivialise 1-bit chemistry discrimination, removing the pressure for complex observers. This would provide an upper bound on $C_{\max}$, but is not formally derived here.

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§8. Limits on Exact Constant Recovery: Underdetermination and the Fano Barrier

T-5 explicitly establishes *bounds* and *order-of-magnitude constraints*, but deliberately avoids deriving raw exact parametric scalars (like 1/137.036) directly from core equations natively.

8.1 The Underdetermination Argument (Derivation Barrier)

The formal reason OPT cannot analytically derive dimensionless standard physical couplings is bounded securely by logical underdetermination. OPT's internal degrees of freedom — $\{C_{\max}, \Delta t, T_{\text{bio}}, q\}$ — are biological and informational quantities with no algebraic path to dimensionless coupling constants such as α or the mass ratios of the Standard Model. The bounds in §§2–5 are therefore the maximum extractable constraints; exact values require additional physical input.

8.2 The Fano Barrier (Identification Precision Barrier)

While underdetermination prevents *deriving* constants, OPT's formalism does place a principled limit on how precisely a bounded observer can *identify* substrate-level laws observationally.

From preprint Eq. (12) — Fano's inequality applied to empirical parameter identification:

$$P(\hat{\theta} \neq \theta) \geq 1 - \frac{T \cdot C_{\max} + 1}{\log_2 N}$$

where N is the number of candidate substrate law hypotheses and T is the observation time. For the fine-structure constant α encoded to k decimal digits of precision, $N \sim 10^k$. For $k = 6$ (precision of $\alpha = 1/137.036$): $N \sim 10^6 \approx 2^{20}$.

The probability of empirically identifying α to 6 decimal places via observation approaches 1 iff:

$$T \cdot C_{\max} \gg \log_2(10^6) \approx 20 \text{ bits}$$

With $C_{\max} = 10$ bits/s: $T \gg 2$ seconds of observation. This is computationally trivial, natively predicting physics experiments cleanly discovering empirical coefficients flawlessly.

However, correctly structurally mapping and successfully explicitly testing *which* of the $\sim 10^{500}$ string landscape vacua we occupy fundamentally requires resolving empirically:

$$T \gg \frac{10^{500}}{C_{\max}} \approx 10^{499} \text{ seconds}$$

— vastly exceeding the universe's age. (Note: The 10^{500} figure is imported from string theory as an illustrative upper bound on possible physical completions. OPT's own Fano barrier applies to the narrower question of empirically distinguishing between OPT-compatible codec configurations — a problem whose N is not yet characterised.) This is OPT's formal restatement of Mathematical Saturation: no $C_{\max}$-bounded observer can empirically confirm which element of a landscape of size $\gg 2^{T \cdot C_{\max}}$ they occupy within a finite observation window.

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§9. Closure Summary and Open Edges

T-5 Deliverables

1. **T-5a.1 (Planck alignment mapping — order-of-magnitude).** Leveraging standard physical coefficients $\{c, \hbar, G\}$ identically as empirical inputs alongside assuming an elementary alphabet $q = 2$, the boundary structural formulas align cleanly bounding $l_{\text{codec}} \approx 1.67 l_P$.
2. **T-5a.2 (Λ upper bound — CLOSED).** $\Lambda \leq 12\pi^2 C_{\max}^2/c^2 \approx 6.3 \times 10^{-15}$ m^{-2}. Observed Λ universally satisfied smoothly.
3. **T-5b.1 (α lower heuristic bound — novel).** Mapping explicit energy ansatz mapping yields $\alpha \geq \sqrt{8 k_B T_{\text{bio}} \cdot f(h^*)/(m_e c^2)}$. While adopting a specialized physical ansatz parameter scaling vs standard generic limits, it structurally frames the constant dependencies explicitly.
4. **T-5b.2 (G upper bound — CLOSED).** $G < R_{\text{obs}}^3/(M_{\text{obs}} \Delta t^2) \approx 9.8 \times 10^{-2}$. Observed G satisfies by 10 orders. Schwarzschild bound: $r_S(\text{brain}) \ll R_{\text{brain}}$ by 25 orders.

5. **Joint $C_{\max}$–α constraint surface (CLOSED - Ansatz Dependent).** The discriminability condition defines a hyperbola cleanly safely in $(\alpha, C_{\max})$ space functionally. Our universe sits comfortably inside the appropriately heuristically allowed region.

6. **Fano barrier & Underdetermination (CLOSED).** Exact derivation of $\alpha = 1/137.036$ from OPT's internal parameters is formally impossible by underdetermination (§8.1). Empirical identification to any finite precision k is achievable once $T \cdot C_{\max} \gg \log_2(10^k)$, which is easily satisfied at the precision of current measurements (§8.2).

Remaining open items within T-5

- **Strong coupling constant α_s.** A lower bound analogous to T-5b.1 for α_s requires the codec to represent nuclear binding. The constraint is $\alpha_s \geq \alpha_{s,\min}(T_{\mathrm{QCD}}, h^*)$ where $T_{\mathrm{QCD}} \sim 200$ MeV is the QCD scale. This bound is straightforward to derive but requires the hadronic mass spectrum as additional input.

- **Upper bound on α from non-relativistic regime.** For the codec to represent atomic physics without full Dirac spinor complexity, $\alpha < \alpha_{\max}$ where $\alpha_{\max}$ is set by the requirement $K(\text{Dirac corrections}) \leq B_{\max}$. This requires a more detailed codec complexity model.

- **Recovering α to higher precision.** The Fano barrier prevents exact derivation, but OPT may narrow the allowed range further by requiring the MDL-optimal coupling — the value of α minimising $L_T(\mathrm{OPT})$ over the joint $(\alpha, C_{\max})$ constraint surface. This requires solving the MDL optimisation numerically once T-5a.1's codec is fully identified with the Standard Model.

This appendix is maintained alongside theoretical_roadmap.pdf. References: Bekenstein (1981) [40], Gibbons-Hawking (1977), Barrow-Tipler (1986) [4], Rees (1999) [5], Verlinde (2011) [38], T-1 through T-4 (this series).