Ordered Patch Theory

Appendix T-2: Deriving General Relativity via Entropic Gravity

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Original Task T-2: Deriving General Relativity via Entropic Gravity Problem: The preprint describes gravity conceptually as "rendering cost" across the Markov Blanket, but does not deploy the available mathematics. **Deliverable:** A formal derivation replacing heuristic gravitational claims with Verlinde's exact mathematical mechanism.

Closure status: PARTIALLY RESOLVED (structural correspondence confirmed; formal derivation open). This appendix establishes the target structural mapping required by T-2. It replaces the heuristic gravitational sketch in preprint §7.2 with Verlinde's exact mechanism, recast in OPT's codec language. It establishes strong correspondences for rendering entropy, Newton's law, and the Einstein field equations. However, several load-bearing bridging assumptions are required (importing the Unruh formula, the Einstein-Hilbert functional, and the stationary ergodic equilibrium), rendering this a structural mapping rather than a closed derivation.

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§1. Rendering Entropy — Formal Definition

The informal concept of rendering cost in §7.2 of the preprint is formalised here as rendering entropy, grounded in the area law established in §3.4 via the predictive cut entropy $S_{\text{cut}}(A)$.

1.1 Definition

Let $A \subset V$ be an observer patch on the substrate graph G, with boundary shell $\partial_R A$. The **rendering entropy** $S_{\text{render}}(A, t)$ is formally defined as the boundary mutual information between the patch and the exterior:

$$S_{\text{render}}(A, t) := I\Big(X_{\partial_R A}\,;\,X_{V \setminus A}\Big)$$

If we assume the latent state Z_t acts as a sufficient statistic capable of capturing exactly the information $X_{V\setminus A}$ reveals about $X_{\partial_R A}$, we posit this boundary correlation converges structurally to the codec's internal conditional uncertainty: $S_{\text{render}}(A,t) \sim H(X_{\partial_R A} \mid Z_t)$. The area bound follows from the structural Markov screening condition $X_{A^\circ} \perp X_{V\setminus A} \mid X_{\partial_R A}$ established in §3.4 (preprint Eq. 7–8):

$$S_{\text{render}}(A,t) \leq |\partial_R A| \cdot \log q =: S_{\max}(A)$$

where q is the alphabet size of the local state space and $|\partial_R A|$ is the number of boundary sites. If the substrate graph approximates a d-dimensional lattice, $|\partial_R A| \sim \text{Area}(\partial A)$, confirming that S_{render} is an area quantity, not a volume quantity.

1.2 Local Rendering Entropy Density

For a continuous approximation (valid at scales much larger than the lattice spacing $l_{\text{codec}} = 1/\sqrt{C_{\max}}$ — noting l_{codec} remains formally uninterpreted dimensionally as a spatial length until the explicit scaling identification in T-5):

$$S_{\text{render}}(A) = \int_{\partial A} s(x)\, dA$$

where $s(x)$ [bits/area] is the **local rendering entropy density** at boundary point x. In the absence of sources, $s(x) = (\log q)/l_{\text{codec}}^2$ is uniform. A local concentration of predictive charge (see §2) perturbs $s(x)$ away from this ground state, generating the entropy gradient that drives the entropic force.

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§2. Predictive Charge — The Codec Analogue of Mass

In Verlinde's framework, mass M enters through the equipartition theorem applied to the holographic screen. OPT requires a codec-theoretic counterpart that is independently defined before any gravitational claim is made.

2.1 Definition

The **predictive charge** Q_M of a source region $M \subset V$ is formally defined purely as the static spatial mutual information between M's internal states and the observer's Markov Blanket boundary over one codec cycle:

$$Q_M := I(X_M\, ;\, X_{\partial_R A})$$

We motivate an analogy to T-1 by mapping $Q_M \approx R_{\text{req}}(h, D_{\min} \mid M) \cdot \Delta t$. This approximation explicitly invokes a massive, unproven **Stationary Ergodic Equilibrium Assumption**: linking the temporal predictive rate ($R_{\text{req}} \cdot \Delta t$) directly to the static spatial boundary correlation (I). The exact conditions for this equality remain an open formal gap. Under this approximation, Q_M

conceptually maps to the number of bits per codec cycle that the source M *forces* onto the observer's boundary representation. This is the informational definition of mass: not inertia, not energy density per se, but mandatory predictive load.

2.2 Proportionality to Inertial Mass

For a macroscopically stable source satisfying the Stability Filter, we assume a direct structural proportionality between the correlation bit-count Q_M and the total energy E_M bound within the region. Avoiding the conflation of static mutual information with active Landauer thermodynamically irreversible erasure limits, we explicitly import the boundary limit defining:

$$E_M = Q_M c_{\text{codec}}^2$$

The proportionality $Q_M \propto M$ — the conventional inertial mass — holds structurally by assuming the standard relativistic correspondence $E_M = Mc^2$ maps externally. This establishes the conceptual bridge from informational codec bounds to standard physics equivalents, deferred formally to an explicit bits-to-mass constant scalar α.

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§3. The OPT–Verlinde Dictionary

Before deploying the mathematics, we make explicit the translation between Verlinde (2011) [38] and OPT. This prevents the derivation from inheriting assumptions of standard entropic gravity that OPT has not earned.

Verlinde (2011)	OPT counterpart	Formal definition in OPT
Holographic screen (area A)	Markov Blanket $\partial_R A$	Boundary of observer patch; derived from locality (§3.4)
Screen entropy $S = A/(4G)$	Rendering entropy S_{render}	$S_{\text{render}} \leq \lvert\partial_R A\rvert \log q$ (§1 above)
Bits on screen N	$N = \lvert\partial_R A\rvert \cdot \log q$	Capacity of boundary representation in codec units
Source mass M	Predictive charge Q_M	$Q_M = I(X_M; X_{\partial_R A})$ (§2)
Test mass m	Test patch load m_p	Predictive charge of displaced test patch
Equipartition $E = \frac{1}{2}Nk_BT$	$E_M = Q_M c_{\text{codec}}^2 = \frac{1}{2}Nk_BT_{\text{codec}}$	Thermodynamic identity at codec boundary
Unruh temperature $T = \hbar a/(2\pi c k_B)$	Codec temperature T_{codec}	$T_{\text{codec}} = \hbar_c \kappa/(2\pi k_B)$ (§4.1)

Verlinde (2011)	OPT counterpart	Formal definition in OPT
Entropic force $F = T\,\Delta S/\Delta x$	Active inference gradient	$F = -\partial\mathcal{F}[q,\theta]/\partial x$ (FEP, preprint Eq. 9)
Newton's law $F = GMm/r^2$	$F_r = -\lambda m Q_M/(4\pi r^2)$	Preprint §7.2 Eq. (15); derived in §4 below
Einstein equations $G_{\mu\nu} = 8\pi G\,T_{\mu\nu}$	Codec curvature equation (§5)	Emerges from Clausius relation on S_{render} (§5)

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§4. Derivation of Newton's Inverse-Square Law

We execute Verlinde's exact three-step mechanism — screen entropy, equipartition, entropic force — entirely within OPT's codec language.

4.1 Codec Surface Gravity and Boundary Temperature

Consider a spherical Markov Blanket of radius r enclosing a source of predictive charge Q_M. At each boundary point $x \in \partial A$, we structurally map the classical scalar potential gradient to the outward entropy gradient, defining the **codec surface gravity**:

$$\kappa(x) := c_{\text{codec}}^2 \cdot \partial_n \log s(x)$$

where c_{codec} is the maximum causal propagation speed in the rendered patch (identified with c in preprint §7.2), and ∂_n is the outward normal derivative.

> **Assumption T-2.A (Radial entropy profile).** The entropy perturbation profile of an isotropic predictive charge Q_M is radially symmetric with gradient proportional to Q_M/r^2. This is structurally equivalent to the Newtonian potential gradient; it is imported as a structural input, not derived from OPT primitives. The subsequent recovery of Newton's law is therefore a conditional derivation contingent on this assumption, not a closed derivation.

Under Assumption T-2.A, an isotropic source Q_M at the origin reduces κ to:

$$\kappa = \frac{Q_M c_{\text{codec}}^2}{4\pi r^2 \cdot s_0}$$

where $s_0 = (\log q)/l_{\text{codec}}^2$ is the ground-state rendering entropy density.

The **codec boundary temperature** is:

$$T_{\text{codec}} = \frac{\hbar_c\,\kappa}{2\pi k_B}$$

where $\hbar_c = 1/C_{\max}$ is the minimum quantum of informational action — the codec analogue of the reduced Planck constant.

4.2 Step 1 — Number of Bits on the Screen

For a spherical boundary of radius r with surface area $4\pi r^2$:

$$N = \frac{4\pi r^2}{l_{\text{codec}}^2} \cdot \log q = S_{\max}(r)$$

4.3 Step 2 — Equipartition Determines T_{codec}

By the equipartition theorem applied to the N independent codec modes on the screen:

$$Q_M c_{\text{codec}}^2 = \tfrac{1}{2} N k_B T_{\text{codec}}$$

Solving for the temperature:

$$T_{\text{codec}} = \frac{2Q_M c_{\text{codec}}^2}{N k_B} = \frac{Q_M c_{\text{codec}}^2 l_{\text{codec}}^2}{2\pi r^2 k_B \log q}$$

> **Consistency Constraint:** Equating this equipartition temperature with the Unruh temperature derived in §4.1 ($T_{\text{codec}}^{\text{Unruh}} = \frac{\hbar_c Q_M c_{\text{codec}}^2 l_{\text{codec}}^2}{8\pi^2 k_B r^2 \log q}$) imposes a strict formal constraint $\hbar_c = 4\pi$. In the natural codec units adopted in §4.5 ($c_{\text{codec}} = 1$), this requires $\hbar_c / l_{\text{codec}}^2 = 4\pi$. In physical units, this is equivalent to the constraint on $C_{\max}$ noted in §7.2, and is resolved in T-5.

4.4 Step 3 — Entropy Change for the Test Patch

A test patch of predictive charge m_p displaced by Δx toward the source changes its overlap with the boundary representation. We explicitly **import the Unruh effect formula** as a structural correspondence at the codec boundary:

$$\Delta S_{\text{render}} = \frac{2\pi k_B m_p c_{\text{codec}}}{\hbar_c} \cdot \Delta x$$

(Note: Because we are importing this Lorentz-symmetry formula rather than deriving it from the lattice, the subsequent force derivation serves strictly as a consistency check of this mapping.)

4.5 Step 4 — The Entropic Force

Verlinde's entropic force formula $F = T_{\text{codec}} \cdot \Delta S_{\text{render}} / \Delta x$ gives:

$$F = T_{\text{codec}} \cdot \frac{2\pi k_B m_p c_{\text{codec}}}{\hbar_c} = \frac{2Q_M c^2_{\text{codec}}}{N k_B} \cdot \frac{2\pi k_B m_p c_{\text{codec}}}{\hbar_c} = \frac{4\pi Q_M m_p c^3_{\text{codec}}}{N \hbar_c}$$

Substituting $N = 4\pi r^2 \log q / l^2_{\text{codec}}$, and substituting $\hbar_c = l^2_{\text{codec}}$ alongside an explicit bits-to-mass dimensional conversion parameter mapping α: α is the bits-to-mass conversion factor with dimensions $[\alpha] = \text{kg/bit}$ (in SI units), to be fixed by the identification $l_{\text{codec}} \to \ell_P$ in T-5.

$$\boxed{F_r \propto -\frac{G_{\text{OPT}}\,(\alpha Q_M)\,(\alpha m_p)}{r^2} \qquad \text{with} \quad G_{\text{OPT}} = \frac{c^2_{\text{codec}}}{\log q}}$$

Restoring the preprint's notation $\lambda = G_{\text{OPT}}/(4\pi)$, this mathematically aligns with **preprint Eq. (15):** $F_r = -\lambda m Q_M/(4\pi r^2)$. Newton's inverse-square law is recovered as a structural correspondence, up to the dimensional conversion factor α^2; its explicit evaluation is deferred to T-5.

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§5. Deriving the Einstein Field Equations

Newton's law (§4) establishes the static, weak-field limit. To recover full general relativity, we follow Jacobson's (1995) thermodynamic method: impose the Clausius relation $\delta Q = T\,\delta S$ on the rendering entropy for every local Rindler-like horizon in the codec.

5.1 Setup — Local Rindler Horizons in the Codec

Consider any point p in the rendered spacetime. The codec's causal structure defines a local Rindler horizon $\mathcal{H}$ — the boundary of the past of a uniformly accelerating observer within the codec. The key ingredients are:

- **Rendering entropy of $\mathcal{H}$:** We formally explicitly import the Bekenstein-Hawking entropy assignment mapping the area law directly:

$$dS_{\text{render}} := \frac{c^3_{\text{codec}}}{4 G_{\text{OPT}}\,\hbar_c}\, dA$$

  Note: This specific coefficient maps the area bound proportionally tracking $S_{render} \propto A$, but the exact numeric constant here is a direct definition imported matching standard physics natively, rather than an algebraic derivation strictly extracted from the pure codec bound.

- **Codec surface gravity κ:** At the local Rindler horizon, $\kappa = c^2_{\text{codec}}/l_{\mathcal{H}}$. The codec temperature is $T_{\text{codec}} = \hbar_c \kappa/(2\pi)$.

- **Heat flux δQ:** The predictive charge flux through dA in proper time $d\tau$ is:

$$\delta Q_{\text{pred}} = T^{\text{pred}}_{\mu\nu}\, k^\mu k^\nu \, dA\, d\tau$$

where $T^{\text{pred}}_{\mu\nu}$ is the predictive stress-energy tensor and k^μ is the null generator of $\mathcal{H}$.

5.2 The Clausius Relation

The Clausius relation $\delta Q_{\text{pred}} = T_{\text{codec}}\,\delta S_{\text{render}}$ applied to every local Rindler horizon gives:

$$T^{\text{pred}}_{\mu\nu}\,k^\mu k^\nu = \frac{c^3_{\text{codec}}}{4\pi G_{\text{OPT}}\,\hbar_c} \cdot \kappa\,\theta_{\mu\nu}k^\mu k^\nu$$

where $\theta_{\mu\nu} = \nabla_\mu k_\nu + \nabla_\nu k_\mu$ is the expansion tensor of the null congruence. To proceed with Jacobson (1995), we must assume that the codec scales structurally satisfying the generic proportional bounds $\delta S_{\text{render}} \propto \delta A$ mapping evenly across all local horizons. Applying the Raychaudhuri equation, the null energy condition $T^{\text{pred}}_{\mu\nu}k^\mu k^\nu \geq 0$, integration over the null surface, and the contracted Bianchi identity:

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} \propto \frac{8\pi G_{\text{OPT}}\,\hbar_c}{c^3_{\text{codec}}}\,T^{\text{pred}}_{\mu\nu}}$$

Subject to the imported Bekenstein-Hawking coefficient (§5.1) and the proportionality assumption $\delta S \propto \delta A$, Jacobson's derivation produces the Einstein field equations in OPT codec language with coupling constant $8\pi G_{\text{OPT}}\hbar_c/c^3_{\text{codec}}$. The cosmological constant Λ arises identically as the mapping constant of integration of the Clausius relation — natively mapping to the ground-state rendering entropy density s_0 tracking the vacuum codec.

The stress-energy tensor $T^{\text{pred}}_{\mu\nu}$ is the *predictive* stress-energy: the distribution of predictive charge density and flux across the rendered spacetime. In the Newtonian limit for pressureless matter, $T^{\text{pred}}_{00} = Q_M/V$ and all other components vanish, recovering §4.

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§6. Gravitational Curvature as Rate-Distortion Overflow

The closure criterion for T-2 requires a formal proof that gravitational curvature is the codec's resistance to rendering information exceeding the rate-distortion equilibrium. §5 provides the Einstein equations; this section makes that identification precise.

6.1 The Rate-Distortion Localization Hypothesis

From T-1, the Stability Filter imposes a global boundary conditional threshold $R_{\text{req}}(D_{\min}) \leq B_{\max} = C_{\max} \cdot \Delta t$. Rate-distortion mappings in AIT are formally global process ensembles. Defining a strictly local predictive constraint requires explicitly extending the formalism (e.g. spatial ergodic sub-ensemble averages), deferred formally to T-5. For the purposes of this structural sketch, we treat local curvature as reflecting the local density of rate-distortion overflow, with the formal justification deferred to T-5.

6.2 Curvature as Codec Resistance — The Formal Identification

To strictly map the rendering entropy bounding functionally mapping $G_{\mu\nu}$, we explicitly construct a formal structural identification matching standard physical gravity actions mathematically natively defining:

$$S_{\text{render}}[g] := \frac{1}{4G_{\text{OPT}}} \int R\sqrt{-g}\, d^4x$$

This is a structural *definition* formally imported exactly matching the Bekenstein-Hawking mapping assigned securely. It is explicitly not algebraically derived tracking directly from T-1 area bounds inherently. Subject to this definition, standard variational calculus gives:

$$\frac{\delta S_{\text{render}}}{\delta g_{\mu\nu}} \propto (G_{\mu\nu} + \Lambda g_{\mu\nu})$$

The Einstein field equations (§5.2) now natively read identically as an optimally bound structural equilibrium:

$$\frac{\delta S_{\text{render}}}{\delta g_{\mu\nu}} \propto \frac{1}{2T_{\text{codec}}} T^{\text{pred}}_{\mu\nu}$$

This defines the **extremal rendering condition**: the metric configuration that minimises the rendering entropy cost given $T^{\text{pred}}_{\mu\nu}$ is exactly the one satisfying Einstein's equations.

Formal statement of the partial closure mapping.

Under this identification, the Einstein tensor $G_{\mu\nu}$ is the metric derivative of the rendering entropy functional. Conceptually, curvature encodes the codec's second-order resistance to metric perturbation: it is large where additional boundary bits must be allocated to accommodate local predictive charge density.

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§7. Event Horizons as Codec Saturation Points

Note: The following analysis treats $R_{req}(p, D_{\min})$ as a well-defined local quantity; this requires the Localization Hypothesis of §6.1 and is therefore heuristic pending T-5.

7.1 The Saturation Condition

An event horizon forms where $R_{\text{req}}(p, D_{\min}) = B_{\max}$ exactly — the boundary at which the Stability Filter is saturated. For a spherically symmetric source of predictive charge Q_M, setting $R_{\text{req}}(r_S) = B_{\max}$ and solving:

$$r_S = \frac{G_{\text{OPT}}\, Q_M}{c_{\text{codec}}^2}$$

This is OPT's native Schwarzschild radius. The standard general-relativistic result is $r_S = 2GM/c^2$, which differs by a factor of 2. This factor-of-2 discrepancy is **not derived** from OPT primitives; matching the classical result would require either $Q_M = 2M$ (an ad-hoc identification) or a proper treatment of the near-horizon geometry that produces the factor naturally. We do not impose this matching; instead, we note the factor-of-2 as an open discrepancy that may be resolved by a full near-horizon analysis.

Inside r_S, $\Delta R(p) > 0$ at every point: the codec is in permanent overflow. The interior of a black hole is the region where the Stability Filter irrecoverably fails — not a location in physical space, but a topological boundary of the codec's representational capacity.

7.2 Hawking Radiation as Codec Boundary Leakage

At the horizon $r = r_S$, the codec temperature with $\kappa = c_{\text{codec}}^4/(4G_{\text{OPT}}Q_M)$ gives:

$$T_H = \frac{\hbar_c\, c_{\text{codec}}^4}{8\pi k_B G_{\text{OPT}}\, Q_M}$$

This reproduces the standard Hawking temperature in structural form. Matching to the physical value requires $\hbar_c c_{\text{codec}}^4/G_{\text{OPT}} = \hbar c^3/G$, which fixes $C_{\max}$ in terms of fundamental constants — introducing a tension with T-1's treatment of $C_{\max}$ as a free empirical parameter. Resolution is deferred to T-5.

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§8. Cosmological Constant as Vacuum Rendering Cost

The cosmological constant Λ appears in §5.2 as the integration constant of the Clausius relation. The vacuum state of the codec is not empty: it is the ground-state configuration of rendering entropy with uniform density $s_0 = (\log q)/l_{\text{codec}}^2$. The associated vacuum predictive stress-energy is:

$$T_{\mu\nu}^{\text{vac}} = -\frac{\Lambda\, c_{\text{codec}}^4}{8\pi G_{\text{OPT}}\, \hbar_c}\, g_{\mu\nu}$$

In OPT, $\Lambda > 0$ corresponds to a de Sitter codec geometry — the codec's ground state is an accelerating expansion. Qualitatively, this is an expected structural rationalization: the Stability Filter preferentially selects configurations where Forward Fan branches are maximally separated (cosmological expansion increases the informational distance between branches, reducing the rate of accidental causal recoupling). This framework provides a qualitative explanation for the sign of Λ, though deriving its extraordinarily small, quantitative observed limits is deferred to the physical constants recovery in T-5.

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§9. Closure Summary and Open Edges

T-2 Deliverables — Partially Resolved (Structural Mapping)

1. **Rendering entropy formalised.** $S_{\text{render}}(A)$ defined via bounding mutual information. Area law confirmed; local density $s(x)$ defined.
2. **Newton's law mapped.** $F_r = -G_{\text{OPT}} Q_M m / r^2$ recovered via Verlinde's mechanism, contingent upon importing the Unruh boundary assumption.
3. **Einstein equations mapped.** $G_{\mu\nu} + \Lambda g_{\mu\nu} \propto T_{\mu\nu}^{\text{pred}}$ aligns with Jacobson's Clausius method, contingent upon horizon-saturation and Einstein-Hilbert functional assumptions.
4. **Closure criterion satisfied as mapping.** $G_{\mu\nu} \propto \delta S_{\text{render}} / \delta g_{\mu\nu}$. Curvature is structurally identified with the metric derivative of rendering entropy — the codec's mapped resistance to rate-distortion overflow. ■
5. **Event horizons.** $r_S = G_{\text{OPT}} Q_M / c_{\text{codec}}^2$ derived as the codec saturation point. Hawking temperature recovered from boundary thermodynamics.

Remaining open edges

- **T-3 (MERA Tensor Networks)** now has a sharper target: the tensor network upgrade of Z_t is required to convert S_{render} from a classical area law into the Ryu-Takayanagi holographic entropy bound. The Jacobson derivation here is the intermediate floor.
- **T-5 (Constants Recovery)** depends on T-2: $G_{\text{OPT}} = c_{\text{codec}}^2 / \log q$ must be matched to the empirical G via the $l_{\text{codec}} \to l_P$ identification. This constrains the codec lattice spacing to the Planck length, providing the first structural inequality for T-5a.
- **Quantum gravity (open):** Deriving the exact Einstein field equations from Active Inference — rather than from Jacobson's thermodynamic method — remains a profound open challenge. The tensor-network upgrade (T-3) and the ADH quantum error correction path (P-2) are the next formal steps.
- **de Sitter extension (open):** The derivation in §5 follows Jacobson and applies cleanly to asymptotically flat and AdS geometries. Extending to dS/CFT — consistent with the observed positive Λ — requires the open mathematical extension noted in preprint §8.3 item 4.

This appendix is maintained as part of the OPT project repository alongside theoretical_roadmap.pdf. References: Verlinde (2011) [38], Jacobson (1995), Bekenstein (1981) [40], Almheiri-Dong-Harlow (2015) [42].