Ordered Patch Theory

Appendix P-2: Conditional Quantum Correspondence via Topological Error Correction

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Original Task P-2: Hilbert Space via Quantum Error Correction Problem: Citing Gleason's Theorem as derivation of the Born Rule is partially circular, as it presupposes Hilbert space geometry without deriving why the predictive space takes that form. **Deliverable:** Analytic derivation showing that the logical qubit structure of a Hilbert space naturally emerges from the codec acting as an error-correcting code.

Closure status: CONDITIONAL CORRESPONDENCE. This appendix maps the bridge from classical information theory to quantum mechanics. It does not natively *derive* quantum fields out of the Ordered Patch Theory (OPT) primitives, but rather establishes a strict **conditional structural correspondence**: mapping exactly what physical properties the OPT codec must satisfy in order for quantum mechanics to emerge from it. We isolate the transition into explicit Bridge Postulates. Under these conditions, the structural homologies mapped in T-3 upgrade to rigorous operator algebraic isometries, enforcing discrete Ryu-Takayanagi limits (P-2d) and isolating the Born rule (P-2e). Deriving these postulates organically from OPT framework physics remains the central open problem of the theory.

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§1. The Algebraic Challenge

Appendix T-3 posited a structural homomorphism between the classical OPT Information Bottleneck algorithm and quantum MERA tensor networks. However, a pure classical stochastic matrix cannot isolate quantum amplitude states or perform unitary operations.

Bridging the boundary between classical capacity bounds and quantum algebra requires mapping the problem functionally. We isolate the conditions required to enforce a partial isometry. Rather than claiming microphysical quantum derivation from classical elements, we trace the precise conditional postulates under which the boundary maps to an algebraic quantum field theory (AQFT) factor and generates error-corrected topological isometries.

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§2. P-2.0: Computational Basis Embedding

Before applying field-theoretic postulates, the discrete OPT classical alphabet $\mathcal{Z}$ must be mathematically mapped into a quantum computational basis.

Bridge Postulate 0 (Computational Basis): The discrete classical states $z \in \mathcal{Z}$ map injectively to an orthonormal computational basis $\{|z\rangle\}$ spanning a target Hilbert space $\mathbb{C}^{\mathcal{X}}$.

Theorem P-2.0: Given Bridge Postulate 0, the classical disentangler permutation matrices $U_\tau \in S_{|\mathcal{Z}|}$ independently lift to exact unitary operators acting on the permutation subgroup of $U(\mathbb{C}^{\mathcal{X}})$.

This condition secures the discrete alphabet structure required to formally evaluate traces in subsequent finite-dimensional steps.

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§3. P-2a: The Bisognano-Wichmann Classification

To treat the codec boundary functionally as an algebraic quantum horizon, strict limits must be met to license the Bisognano-Wichmann classification theorem.

Bridge Postulate 1 (CCR): The Markov Blanket variables at the continuous boundary limit satisfy the Canonical Commutation Relations: $[\phi(x), \pi(y)] = i\hbar\,\delta(x-y)$. (Required to treat the boundary as an operator-valued quantum field).

Bridge Postulate 2 (Rindler Horizon Analogy): The boundary horizon possesses global Lorentz symmetry and operates upon a quantum field in the vacuum state, mathematically analogous to an accelerating Rindler wedge.

Bridge Postulate 3 (Haag-Kastler Limits & Split Property): The bounding algebra of the sequence obeys the AQFT Haag-Kastler net axioms: locality, covariance, and positive spectral energy flow properties. Furthermore, the net satisfies the AQFT split property, establishing local type-I factors that allow restriction to finite-dimensional subspaces.

Theorem P-2a (Conditional Type III$_1$ Factor): Given Bridge Postulates 1, 2, and 3, the Bisognano-Wichmann Theorem (1975) conditionally applies. The modular flow generated maps to a geometric Lorentz boost. Connes classification guarantees the field structuring the horizon acts precisely as a Type III$_1$ von Neumann factor.

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§4. P-2b: Noise-Resilience & ADH Mapping

A globally defined Type III_1 von Neumann factor does not admit standard finite-dimensional trace-class density matrices. To evaluate the bulk-boundary duality established by Almheiri, Dong, and Harlow (ADH), we must restrict the algebra.

Bridge Postulate 4 (Knill-Laflamme Conditions): The classical codec sequence inherently forms a continuous Quantum Error-Correcting Code (QECC) satisfying exact Knill-Laflamme bounds.

Theorem P-2b (Conditional ADH Holography): Given BP 4 and the explicit split-property regularization provided by BP 3, the algebra conditionally restricts into the locally finite-dimensional logical code subspace $\mathcal{C}^{(\tau)}$. Within this restricted subspace, the external boundary noise is filtered through the Knill-Laflamme mappings, recovering local bulk operators on the boundary consistent with the ADH theorem.

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§5. P-2c: Restricted Stinespring Trace Algebra

Resolving data compression mathematically requires identifying the classical coarse-graining step W_τ with the action of the partial isometry MERA adjoint map $w_\tau^\dagger$.

By Stinespring's dilation theorem, a Completely Positive Trace-Preserving (CPTP) map implies there exists a general isometric dilation/recovery structure $V : \mathcal{H}_S \to \mathcal{H}_S \otimes \mathcal{H}_E$. This general existence theorem does not natively identify the OPT classical matrix W_τ as the isometry itself. That identification must be bridged.

Bridge Postulate 5 (Unitary Covariant Noise): The environment noise over the mapped channel evaluates as a strictly unitarily covariant map: $\mathcal{N}(U\rho U^\dagger) = U\,\mathcal{N}(\rho)\,U^\dagger$.

Bridge Postulate 6 (Isometry Identification): The classical coarse-graining matrix W_τ identically translates as the CPTP trace computing over the environment of the exact MERA isometry's adjoint $w_\tau^\dagger$.

Theorem P-2c (Conditional Restricted Isometry): Given BP 4, BP 5, and BP 6, the classical coarse-graining algorithm maps successfully as the adjoint of a partial linear isometry. *Proof approach:* Exact QEC on the restricted code subspace (BP 4) provides general recoverability. Rather than asserting the dilation automatically enforces inner-product equivalency, BP 6 explicitly bridges the gap, postulating that the classical matrix identically translates as the quantum dilation's trace component. Therefore, over the finite-dimensional code subspace, the classical map acts operationally as the adjoint of the target MERA isometry.

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§6. P-2d: Ryu-Takayanagi and Schmidt Rank

The classical OPT framework limits continuous channel capacities mapping the bounds $\chi_{\text{classical}} = 2^{B_0/N}$. To function as a valid exact Hilbert space dimension rather than a continuous effective scale, the target mapping explicitly imposes the integer capacity constraint $2^{B_0/N} \in \mathbb{Z}^+$.

Theorem P-2d (Conditional Ryu-Takayanagi Limit): Given the successful realization of P-2c restricting the operations to exact linear isometries, the classical capacity dimension ($\chi_{\text{classical}}$) formally establishes the quantum Schmidt rank (χ_{quantum}) across the network bonds. This equivalence strictly generates the discrete Ryu-Takayanagi entropy limit.

Proof approach: With the classical matrix conditionally identified as a true partial isometry (P-2c), the dimension of the mapped channel limits the virtual geometric bonds connecting the MERA nodes. In the quantum state, the maximal bipartite entanglement across any topological boundary is explicitly structured by the minimal cut γ_A, with the local Hilbert space dimension at each cut established by the bond's Schmidt rank. Since the bottleneck capacity dictates this rank ($\chi_{\text{classical}} = \chi_{\text{quantum}}$), the geometric entanglement formally bounds strictly across the minimal cuts:

$$S_{\text{vN}}(\rho_A) \leq |\gamma_A| \log \chi_{\text{quantum}}$$

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§7. Topological Coherence and Gleason Traces

Generating the Born Rule requires moving beyond statistical diagonal probabilities and isolating off-diagonal frames $\rho_{zz'}$.

Bridge Postulate 7 (Kochen-Specker Non-Contextuality): The probability assignment associated with a predictive output branch is independent of other mutually co-measurable orthogonal pathways.

Theorem P-2e (Conditional Born Rule Formulation): Given BP 7, and assuming the projective probabilities assigned by the OPT algorithms form complete mathematical frame functions, Gleason's Theorem conditionally derives the Born Rule.

Proof approach: The minimum dimensional limitations established by scaling the finite discrete basis space natively satisfy $\dim(H) \geq 3$. Assuming the probabilistic predictive structures satisfy the mathematical requirements of a completed frame function $\mu(P)$ summing to 1, Gleason's Theorem (1957) states there is only one valid probability measure mapping the space:

$$\mu(P) = \text{tr}(\rho_t \, P)$$

This result generates the Born Rule trace exclusively, validating the probabilistic quantum matrix mapping transition conditionally.

This appendix is maintained as part of the OPT project repository alongside theoretical_roadmap.pdf. References: Almheiri-Dong-Harlow (2015), Takesaki (2003), Holevo (1973), Knill-Laflamme (1997), Gleason (1957), Bisognano-Wichmann (1975).