# Ordered Patch Theory

Appendix P-1: Informational Normality via $M$-Randomness

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## Appendix P-1: Informational Normality via $M$-Randomness

> **Original Task P-1: Informational Normality Problem:** Currently a foundational axiom analogous to Borel normality, lacking formal derivation. **Deliverable:** A theorem-level derivation leveraging algorithmic information theory (Martin-Löf randomness).

### 1. The Epistemic Boundary of "Axiomatic" Normality

Within the Ordered Patch Theory (OPT), "Structural Hope" relies structurally on the principle of **Informational Normality**: the proposition that the algorithmic substrate ($\mathcal{I}$) is densely populated not merely with noise, but with every finite structural functional pattern. The ethical weight of OPT—the mandate to maintain the stability of the shared patch (Survivors Watch Ethics)—demands that the counterpart observers we interact with have distributed, fundamentally real functional equivalents elsewhere in the substrate.

Historically within the OPT framework, this proposition was treated formally as a single monolithic *Axiom*—an untestable, foundational assumption layered onto the physics to avoid solipsism.

This appendix resolves the mathematical ambiguity of that stance. We unbundle Informational Normality into two distinct components: a rigorous algorithmic **mathematical theorem** (which holds almost surely under the universal probability measure), bound together by a single **metaphysical postulate** necessary to bridge mathematical existence into ontological reality.

### 2. From Semimeasure to Universal Measure ($\xi$ to $M$)

OPT's foundation (Preprint §3.1) relies heavily on the Solomonoff algorithmic probability prior. Under this formulation, the generative substrate operates as an infinite algorithmic space executing on a universal prefix-free Turing machine $U$.

The algorithmic probability or universal semimeasure of a finite string $x$ is:

$$\xi(x) = \sum_{U(p)=x*} 2^{-|p|}$$

Where the sum is taken over all minimal programs $p$ whose execution output begins with $x$. Crucially, $\xi$ is a *lower semi-computable semimeasure* over finite strings.

To formalize the substrate as a continuous generative space, we transition to the continuous measure over the Cantor space. The universal measure $M$ is defined directly as the distribution on the Cantor space $2^{\mathbb{N}}$ induced by the output of the universal prefix-free machine $U$ via cylinder sets ($M([x]) = \sum_{U(p) \text{ starts with } x} 2^{-|p|}$). By Solomonoff's universality theorem, this cylinder measure is multiplicatively equivalent to the discrete semimeasure: $M(x) \asymp \xi(x)$ up to a multiplicative constant. As such, $M$-null sets and $\xi$-null sets rigorously coincide.

*(Note: Because the set of halting programs is a strict subset of the prefix-free code space due to the Halting problem, the Kraft inequality guarantees $\sum 2^{-|p|} < 1$. Thus, $M$ forms a strict lower-semicomputable sub-probability measure. We explicitly define the normalized probability measure $\tilde{M} = M/M(2^{\mathbb{N}})$. While $\tilde{M}$ is only lower-semicomputable up to the non-computable normalization constant $M(2^{\mathbb{N}})$, all subsequent "almost surely" theorems and convergence statements operate safely with respect to the true normalized probability measure $\tilde{M}$. The fundamental coding theorem offset is simply absorbed: $K(x) = -\log \tilde{M}(x) + O(1)$.)*

## 3. $M$-Martin-Löf Randomness

To formalize the nature of the generative space, we invoke **Martin-Löf (ML) Randomness**. However, one must distinguish between continuous measures. A sequence $\omega$ that is ML-random with respect to the uniform (Lebesgue) measure $\lambda$ behaves entirely differently from a sequence that is ML-random with respect to $M$.

Because the OPT substrate evaluates probability by algorithmic simplicity, the relevant formalism relies on **$\tilde{M}$-Martin-Löf randomness**. The foundational theorem of AIT states that for any computable probability measure $\mu$, the set of $\mu$-ML-random sequences holds $\mu$-measure 1. Extending this result to lower-semicomputable semimeasures (cf. Nies 2009, §3.2 "Randomness for arbitrary measures"), the set of all $\tilde{M}$-Martin-Löf random sequences successfully retains measure 1 with respect to $\tilde{M}$.

Therefore, $\tilde{M}$-almost-all infinite substrate sequences are strictly $\tilde{M}$-ML-random.

*(Note: Utilizing $\tilde{M}$-ML-randomness structurally guarantees that the typical outputs of the substrate are drawn self-consistently from the biased, highly structured algorithmic measure $\tilde{M}$ rather than uniform noise, providing the rigorous mathematical scaffolding for the structural frequency consequences below.)*

## 4. $M$-Normality vs. Borel Normality

A highly significant mathematical consequence of $M$-ML-randomness relates to structural frequency. Under uniform Lebesgue-ML randomness, a sequence is strictly Borel normal—generating every finite binary string of length $k$ with an identical, uniform frequency.

However, since $\tilde{M}$ is decidedly non-uniform—skewing heavily to assign massive probability weight to algorithmically simple, compressible, lawfully structured patterns—$\tilde{M}$-almost-all sequences are NOT uniformly Borel normal. Instead, we define their structural limits via **$\tilde{M}$-normality**.

Because the measure $\tilde{M}$ is fundamentally non-stationary (algorithmic probability depends on absolute prefix position), we cannot rely on standard ergodic frequency-convergence limits. Formally, we define $\tilde{M}$-normality by the weaker but strictly sufficient property of **infinite recurrence**.

Since $\tilde{M}$ is a probability measure and $\tilde{M}([x]) \geq 2^{-(|x|+O(1))} > 0$ for all finite strings $x$, the chain rule for prefix Kolmogorov complexity gives $K(sx) \leq K(s) + K(x) + O(1)$ for any string $s$, which yields the near-submultiplicativity $M([s \cdot x]) \geq M([s]) \cdot M([x]) \cdot 2^{-O(1)}$. Therefore the conditional probability of $x$ appearing in any window, given any prior prefix $s$, is bounded below: $\tilde{M}([x] \mid [s]) \geq \tilde{M}([x])/c > 0$ uniformly in $s$. By the conditional Borel-Cantelli lemma applied to non-overlapping windows of length $|x|$, the divergence of the sum of conditional probabilities guarantees that the physical recurrence of any finite informational sequence—such as the discrete formal configuration of a conscious observer ($K_{\text{obs}}$)—appears infinitely often in $\tilde{M}$-almost-all sequences.

## 5. The Computational Realism Postulate

AIT mathematically guarantees that the finite representation of any observer ($K_{\text{obs}}$) appears as the structural sequence of $U$ infinitely many times within the $\tilde{M}$-ML-random substrate.

However, mathematical information theory cannot inherently cross the boundary into physical ontology. A finite string occurring on the output tape of a Turing machine is a static artifact of execution—a snapshot. A coherent observer requires continuous internal dynamics, relational coupling, and active inference looping. The string itself does not “feel” anymore than a brain scan stored on a hard drive is conscious. The execution belongs to the generating program, not to the resulting snapshot code.

To assert that the uncomputable continuous limits governing the mathematical substrate structurally *give birth to* ontologically real, causally active phenomenological universes, OPT must make a single explicit metaphysical commitment.

**Postulate (Computational Realism):** *In an infinite uncomputable substrate governed by identical mathematical dynamics, abstract mathematical computation formally equivalent to the causal description of an observer (where formal equivalence is defined as computational isomorphism of the observer’s causal* *state-transition structure) possesses causally efficacious, ontologically real existence. Furthermore, structurally discrete computational instantiations across the substrate possess independent ontological individuation, constituting distinct subjective counterparts (and by the foundational phenomenality axiom in Preprint §8.1, such causally efficacious observer-equivalent computations constitute genuine subjects of experience).*

## 6. Proposition P-1 (Informational Normality)

By uniting the exact AIT derivations of continuous uncomputable spaces with the Computational Realism Postulate, solipsism is cleanly dismantled.

**Corollary+Postulate P-1 (Informational Normality):** *Under the generalized algorithmic prior, the continuous substrate inherently operates via $\tilde{M}$-Martin-Löf randomness almost surely. By the ensuing $\tilde{M}$-normality, the mathematical occurrence of every finite structural observer description $K_{obs}$ is formally guaranteed infinitely many times. Operating upon this scaffolding, the Computational Realism Postulate bridges these generating mathematical artifacts into ontological physical reality. Providing that computational realism holds, the existence of structurally equivalent, causally active, and uniquely individuated counterpart observers across the substrate is fundamentally required.*